Time-Experience and the Limits of Objectification: Remarks on the Phenomenological Implications of Hermann Weyl's Treatment of the Intuitive Continuum

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Time-Experience and the Limits of Objectification: Remarks on the Phenomenological Implications of Hermann Weyl's Treatment of the Intuitive Continuum

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Résumé : L'article se concentre sur le problème de l'expression mathématique du continuum intuitif de l'espace et du temps, en suivant certaines étapes significatives de la pensée de Hermann Weyl sur le sujet du point de vue de ses relations avec la phénoménologie de Husserl. Si Weyl et Husserl sont d'accord pour reconnaître l'irréductibilité d'un point de référence subjectif, leur convergence pour reconnaître le rôle central de l'expérience intuitive est également frappante. L'échec de la tentative de Weyl dans *Das Kontinuum* (1918) de fournir une formulation mathématique pour le continuum intuitif, dû à la reconnaissance d'une hétérogénéité radicale entre le caractère fluide de ce dernier et le continuum mathématique, est pris en compte. Dans la période intuitionniste-brouwerienne qui suit, cependant, Weyl développe un schéma formel basé sur la relation partie-tout qui est capable de capturer le caractère dynamique du continuum intuitif dans lequel les éléments individuels deviennent de simples limites. Dans ce contexte, les relations entre la conception de Weyl du continuum et les analyses husserliennes du temps sont discutées.

Abstract: The paper focuses on providing a mathematical expression for the intuitive continuum of space and time, tracing some significant stages of Hermann Weyl's thinking on the subject from the standpoint of its relations to Husserl's phenomenology. While Weyl and Husserl both recognise the irreducibility of a subjective point of reference, their agreement in recognising the central role of intuitive experience is also striking. The failure of Weyl's attempt in *Das Kontinuum* (1918) to provide a mathematical formulation for the intuitive continuum, due to the recognition of a radical heterogeneity

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between the fluid character of the latter and the mathematical continuum, is taken into account. In the subsequent intuitionistic-Brouwerian period, however, Weyl develops a formal scheme based on the part-whole relation that is capable of capturing the dynamic character of the intuitive continuum, in which individual elements become simple boundaries. In this context, the relations between Weyl's conception of the continuum and Husserlian analyses of time are discussed.

1 Introduction

The objective of this contribution is to elucidate the essential elements of Hermann Weyl's perspective on the formal treatment of the intuitive continuum and to demonstrate its intrinsic phenomenological significance. This is done within the broader framework of the discourse on the problem of objectification and intuition and with reference to Husserl's analysis of time consciousness, a topic that appears to be absent from the current research literature.

It is undeniable that at least many of Weyl's decisive contributions to mathematics and physics are successfully interwoven with philosophical considerations in line with important aspects of the phenomenological project.[1] This makes Weyl, one of the leading scientists of his time and a non-superficial reader of Edmund Husserl,[2] an exemplary case of a mathematician-scientist who is aware of the assumptions and limits of the scientific attitude, while at the same time confirming the fruitfulness of a phenomenological philosophy

1. Husserl's influence on Weyl, especially in the period 1918-1922, was considerable [Bell 2004, 173], and is well known. Weyl, whose wife had been a student of Husserl in Göttingen, acknowledged that it was Husserl who led him away from his earlier positivism towards a freer world view [Weyl 1954, 637]. Weyl himself maintained that first his predicativist programme in *Das Kontinuum*, then his adhesion to Brouwer's intuitionism, were influenced by Husserl's phenomenology, and in particular by the epistemology present in the *Logical Investigations* and *Ideas I*. Weyl's distancing from Husserl can be traced back to the early 1920s [Marion 2004, 129]. In *Erkenntnis und Besinnung* [1954], in which Weyl retraces his intellectual journey, he does not neglect to express his reservations about the father of phenomenology and the points in which he considers the Husserlian approach to be inadequate. This explains Weyl's turn to other philosophical orientations, from Cassirer to existentialism and religious mysticism [see Bell 2004].

2. The introduction to his work *Raum, Zeit, Materie—Vorlesungen über Allgemeine Relativitätstheorie* [Weyl 1970] contains a surprisingly faithful exposition of some of the main themes of Husserl's *Ideen I*. Regarding Weyl's relationship to Husserl's phenomenology, at least the following texts are worth mentioning, in addition to the other references given in this paper: [Bernard 2013], [Mancosu & Ryckman 2002, Ryckman 2003, 2005], [Sieroka 2019]. See Scholz [2001] for a general presentation of Weyl's scientific work.

that seeks to establish a productive dialogue with the sciences. In our examination of certain philosophical aspects of Hermann Weyl's reflections on topics related to the mathematization of the intuitive continuum, with reference to a number of points of phenomenological and, in particular, Husserlian provenance, we shall concentrate on Weyl's writings of the 1910s and 1920s; it should be noted at the outset that at that time he could not have been familiar with the *Krisis* [Husserl 1954], which had not yet been published, nor with most of Husserl's writings on temporalities (which we shall consider later), which have only recently been published. This, of course, makes it all the more significant that the two authors seem to converge on relevant points, albeit in different ways and from different perspectives. In any case, we will try from time to time to detect points of contact with positions present in Husserl's phenomenology, not so much by a reconstructive analysis of the relations between the two authors, but by showing the phenomenologically relevant implications of some of Weyl's decisive theses from a systematic point of view, sometimes even beyond Weyl's direct knowledge of Husserl's texts.

2 The quest for objectification

The question of the scientific objectification of the pre-scientific world and its subjective character will provide us with an opportune starting point for dealing with the continuum problem as treated by Weyl. The tendency which, as is well known, has imposed itself on the scientific enterprise at least since the time of Galileo, is to eliminate any vestige of subjective variability. The typical leitmotif of scientific modernity is the objectivist ideal, according to which any consideration that aspires to be truly scientific must be translated into the formal language of mathematics. What is objective and scientifically relevant must therefore be expressed in purely quantitative terms. The *intuitive* and qualitative component at the pre-scientific basis of scientific experience would have to be eradicated by the objectifying enterprise. The overcoming of the naively immediate and manifest image of the world as an object of direct experience, and its translation into purely objective and formalizable terms, will thus have to take place through the overcoming of the intuitive moment as constitutive of pre-scientific life.

Even at the pre-scientific level, the hallmark of objectivity is constancy across the various subjective modes of givenness.[3] In perceptual experience, for example, the transcendence of the perceived object is guaranteed by the permanence of its identity in the variation of its modes of appearance to the perceiving subject, which are always perspectival and therefore one-sided. In general terms, objectivity—and thus the character of mental independence—is linked to the exhibition of structures that remain invariant while the subjective access to them constantly varies. It is precisely on the basis of this dialectic

3. For this and the following discussion I refer to Wiltsche [2021].

between variation and invariance that the so-called *secondary properties* are identified and qualified as merely subjective. The truly objective properties, the *primary properties*—such as the specific shape of something—are those that show relations that remain invariant as the subjective perspective changes, and are therefore not reducible to any of these [Wiltsche 2021, 471].

Physics takes this need to isolate structural invariants to an extreme limit of formalisation. It replaces the concrete subject with an abstract coordinate system, while variability is translated into the terms of transformation laws, so that physical objectivity is accorded only to those properties that remain invariant under a given group of transformations. The functional relationship between these properties is then determined quantitatively in terms of laws. This makes it possible to define objectivity in terms of the requirement that the transformation of laws must be covariant, i.e., that their mathematical form must be preserved in the transformation from one reference system to another. In this way, physics should be able to remove all traces of subjectivity from its representation of reality and achieve a *view from nowhere*.[4] As Weyl notes in *Philosophy of Mathematics and the Natural Science* [1949]—the first German version of which dates from 1927—a tension towards the representation of reality by mathematical symbols is immanent in the development of physics. This tension is guided by the regulative ideal of complete symbolisation and thus the systematic exclusion of any subjective element in physical theoretical thought [Weyl 1949, 110 *ff.*], [Wiltsche 2021, 473].

3 On the non-objectifiable residue

For Weyl, however, this ideal of pure symbolic objectification is destined to be unattainable:

> All knowledge, while it starts with intuitive description, tends toward symbolic construction. No serious difficulty is encountered as long as one deals with a domain consisting of a finite number of points only, which can be "called up" one after the other. The problem becomes a serious one when the point-field is infinite, in particular when it is a continuum. A conceptual fixation of points by labels [...] that would enable one to reconstruct any point when it has been lost, is here possible only in relation to a *coordinate system*, or frame of reference. [Weyl 1949, 75]

The fundamental point here is that this reference system

> has to be exhibited by an individual demonstrative act. The objectification, by elimination of the ego and its immediate life of intuition, does not fully succeed, and the coordinate system

4. For all this see [Wiltsche 2021, 471 *f.*], who also refers to Cassirer [1953].

> remains as the necessary residue of the ego-extinction.[5] [Weyl 1949, 75]

Now, this failed attempt to annihilate the ego is somewhat reminiscent of Husserl's well-known mental experiment concerning the annihilation of the world [Husserl 1976, § 49]. The fundamental difference is that Weyl reverses the procedure [Wiltsche 2021, 475]: instead of only exposing the world to annihilation, he also tries to imagine the annihilation of the subject itself [Sieroka 2019, 113].[6] In both cases, the result of the process is a subjective residue that resists the attempted annihilation.

For Weyl, therefore, any project of objectification must recognise the necessity of a residual component that cannot be objectified. The latter corresponds to the *zero point* of the *coordinate system* which objectification presupposes and which, as a condition for it, clearly cannot be subjected to objectifying formalisation. Even in the most formalised representations of physics, then, we find again the "transcendental" necessity of a subjective element, as an extremely idealised form of the living body (as *Leib*) as the zero point of spatial orientation, which Husserl discusses at length in the lectures *Ding und Raum* (1907) [1973b]. It is from this point of reference that the kinaesthetic series with their noematic correlates depart, giving rise to the experience of a lived space with its objectual units [Husserl 1973b, 131, 156], [Lobo 2019, 92]. It is this necessarily subjective rest that modern science, for Weyl, is unable to eliminate. Indeed, objectification implies that no point in space is privileged over the others, but this poses the problem of its identification when we try to establish the link between mathematical formalism and empirical data [Wiltsche 2021, 474]. When it comes to identifying a point from a continuous milieu of points, this is only possible by introducing a coordinate system [see Weyl 1970, 8]. It is precisely the necessary recourse to a reference system that implies the reintroduction of subjectivity into the purely symbolic image of reality produced by physics.[7] In particular, the origin of the coordinate system corresponds to the subjective zero point of spatial orientation, as it can be found in [Husserl 1973a, 116 *f.*], [1973b, 131, 280], [Wiltsche 2021, 475]. The coordinate system, as an instrument and condition of the formalisation by which physics seeks to achieve its ideal of objectivity, is thus an eminent case of the attempt to root the idealities employed by the sciences in the sphere of pre-scientific experience, with its

5. A similar formulation can be found in several of Weyl's writings.

6. However, by speaking of an "individual subject", Sieroka introduces some ambiguity here, for he seems to be referring to the empirical subject, which Husserl himself submits to the procedure of annihilation, since this subject is part of the world.

7. This contribution will not consider the coordinate-free differential-geometric approaches that gained prominence in the foundation of physics, as this would diverge from the topic under discussion and depart from the historical context delineated here. For a discussion of these approaches, see [Wallace 2019].

embodied perspective [Wiltsche 2021, 475]. Interestingly, this shows that Weyl was working on such issues independently of Husserl's *Krisis*.[8]

From all this it is already clear that the attempt to translate physical reality and its space-time continuum into a formalised conceptual scheme does not completely remove the intuitive and thus unformalized content of this continuum. It is true that points in space and time can be translated into geometric terms and objectively fixed by a coordinate system, but this system itself cannot be derived from geometric axioms, but must be given by an *individual intuitive act* [Ryckman 2005, 131, 134]. The limit of scientific objectification lies in the fact that reality, even the most mathematised, has already been constituted by consciousness, and only for this reason can it be mathematised. The exact conceptual definition of points in time according to the model of the real numbers presupposes a one-dimensional axis, the orientation of which is not determined conceptually but intuitively. Therefore, in line with the phenomenological point of view, the mathematical geometric world, free of all sensory qualities, contains the imprint of its origin in what is given in intuition,[9] which is relative to a subjective perspective [Ryckman 2005, 131].

8. A notable difference is that for Husserl, objectification through symbolic activity can lead the sciences to a loss of meaning, whereas the result of Weyl's reflection is that symbolisation is, after all, the only means that science has to escape the factuality, immediacy and subjectivity of perception [da Silva 2017, 322]. For Weyl, if it is true that intuition is the starting point, it is also true that science must ultimately deal with a sphere in which knowledge must necessarily rely on symbolic construction [Weyl 1932, 80], [Weyl 1949, 75].

9. However, if one wishes to draw a parallel with Husserl on this point, one must bear in mind that Husserl's notion of intuition is much broader, extending far beyond immediate sensible experience to include, for example, the field of *categorial* objects. With regard to mathematics, Weyl does not seem to acknowledge that phenomenological intuition goes beyond the possibilities of intuition as understood by the intuitionist orientation in the philosophy of mathematics [Mancosu & Ryckman 2002, 149 *ff.*], [da Silva 2017, 320]. This discrepancy is confirmed by Weyl's reservations about the possibility of *essence intuition* [da Silva 2017, 320 *f.*]. Husserl's influence, which Weyl explicitly recognised [Weyl 1918, IV], may rather consist in Husserl's emphasis on the epistemological primacy of *evidence* [Ryckman 2005, 109] and the "given", which has little to do with the radicality of Weyl's position in the period of his commitment to Brouwerian intuitionism [da Silva 2017, 318], and does not necessarily lead to such a position [da Silva 2017, 347]. According to da Silva, in his letter to Weyl of 10 April 1918, Husserl certainly expresses his appreciation for *Das Kontinuum* and the necessity—expressed therein—of a return to logical-mathematical intuition for foundational purposes [see Van Dalen 1984, 3], but he also makes it clear that not all mathematics is suitable for treatment by intuition (this is the case with purely formal mathematics) [da Silva 2017, 347]. In any case, for Weyl, recourse to intuition and the non-argumentative form of rational justification that it entails was a way out of his earlier conventionalism [da Silva 1997, 282 *f.*].

4 On the irreducible gap between the intuitive and the formal continuum

The subjective relativity of the representation of something in intuition as opposed to conceptual determination is thus a central epistemological problem for Weyl [Ryckman 2005, 135 *f.*], which is related to his reflections on the nature of the *intuitive continuum* and the possibility of its mathematical treatment, which is the topic we are now to deal with.

That a continuity can be derived from discrete definite atomic elements belongs to the classical position of set theory, whereby the problem of the continuity of time (and space) is then subordinated to the issue of the arithmetic, numerical continuum. We cannot go into detail here, but suffice it to say that, according to Georg Cantor, a set is considered continuous only if equivalence with the set of real numbers is possible, so that every point in the continuum corresponds to a point in the set of real numbers and vice versa [Cantor 1932, 192], which presupposes (against Aristotle) the admission of actual infinity. The position of the independent existence of all possible subsets of the set of real numbers is thus assumed a priori, independently of any process of construction [Weyl 1925, 518]. What is at stake, then, is the conviction that the continuum consists of an *actual infinite set of points*, existing in themselves, from which infinite subsets can be cut out—which necessarily exist because the whole from which they are extracted exists itself [Pradelle 2019, 183]. One can thus understand Cantor's doctrine of the continuum as an attempt to reduce continuous manifolds to numerical manifolds (of discrete individual elements) and thus to make them accessible to a purely symbolic-quantitative treatment. (This assumption became standard after Cantor, and even today there is a tendency to identify a linear continuum with the set of real numbers [Bedürftig & Murawski 2010, 156].)

This is acceptable for a purely arithmetical continuum, such as the series of real numbers, which can be said to consist of well-defined individuals. From a philosophical point of view, however, we run into difficulties when we try to carry out what could be provocatively called an "Arithmetisierung der Geometrie" [Husserl 1954, 44], by imposing this formal arithmetic model on the continuum of time and space, although the latter contains an intuitive (in the phenomenological sense of *anschaulich*) and therefore non-formalizable component. It is no coincidence that Husserl included the pure theories of time and space (geometry) in the disciplines of the synthetic a priori and not in the analytic-formal a priori, precisely because they contain an ineradicable intuitive-material element that cannot be formalised, and for essential reasons. As Oskar Becker points out, "although the material-eidetic laws are of course a priori, they are not rationally intelligible in the strict sense: they are contingent-aprioric" [1923, 393, my translation]. It is therefore not possible to completely "rationalise" the intuitive and inexact ("morphological") continuum of time and space by modelling it on an arithmetical ("ideal") continuum made up of elements defined with absolute exactness,[10] because the latter is atomistic in nature and can be traced back to a precisely defined set of pre-existing points,[11] which does not take into account the specific nature of the continuum of space and time.

This is exactly what also Weyl tried to do in *Das Kontinuum* [1918]—albeit from a predicativist rather than a set-theoretic perspective[12]—only to realise in the end that such a programme is doomed to failure precisely because of the insurmountable discrepancy between the mathematical continuum of numbers, even real numbers, and the intuitive continuum of space and time.

> In particular, phenomenal time contains nothing corresponding to the immediately evidenced relation between a discretely given number x and its unique successor that underlies the construction of the purely predicative mathematical continuum. [Ryckman 2005, 130]

The irreducibility of the intuitive continuum to the mathematical one is due to the fact that the latter is *analytic* in nature, consisting of a series of individual elements which give rise to it—at least when their connectedness is explicitly stated or proved—whereas the former cannot be derived by putting together a series of individuals, even if this series is "dense". Individual points in the intuited space and time do not have an autonomous and substantial existence; there is no isolated and independent point within the flow of time or the extension of space, where each element is a "transit point" in the flow, passing through a continuous *synthesis* from one point to another. Therefore, the *fluid* nature of the intuitive temporal and spatial continuum prohibits fixing anything exactly, but allows only *approximation* [Weyl 1918, 70]: there are only fuzzy partitions in morphological continuums, so that we cannot specify

10. On the distinction between ideal and morphological concepts, see [Husserl 1976, § 74].

11. See Dedekind's principle [Dedekind 1960, § 3; see also § 5]. Dedekind formulates this principle on the basis of the spatial, phenomenal representation of a line, and then shows that a correspondence can be established between series of line points and classes of rational numbers, insofar as the ordering relations in both cases have the same formal properties. Nevertheless, it seems to me that his approach clearly leads to a formal solution that overshadows the dynamic and fluid character of the intuitive continuum, assuming the fixity of the points on the line.

12. In *Das Kontinuum*, Weyl critiques the impredicative method of classical analysis, on the basis of his rejection of the conception of the mathematical object as existing independently of consciousness, which leads to the paradoxes of set theory [Ryckman 2005, 109]. Opposing the epistemological Platonism of set theory, Weyl proposes a predicative analysis that takes the sequence of natural numbers as its basic object. The underlying primitive relation, whereby y is the immediate successor of x, is for Weyl accessible to pure intuition. In this way it would be possible to provide a solid foundation for mathematics [Ryckman 2005, 129].

exactly what belongs to any part of the continuum and what does not [Becker 1923, § 1 B, 421 *f.*].

5 A new model for bridging the gap

But Weyl would not stop at this impossibility of reconciling the intuitive continuum and the arithmetic continuum.[13] Rather, he will later develop tools capable of treating the space-time continuum mathematically, but in a more adequate way than set theory.[14] In his [1921] text "Über die neue Grundlagenkrise der Mathematik", in which Brouwer's influence is evident,[15] Weyl makes a more precise distinction between a "discrete" or "atomistic" continuum and a "continuous" one. The former consists of a series of real numbers as well-defined individuals. At this point Weyl has definitely moved away from any atomistic theory of the continuum and even from his earlier predicativist approach in *Das Kontinuum*. Following Brouwer, he replaces the realism of actual infinity in set theory with the "idealistic" conception of the

13. For the following discussion, see [Bell & Korté 2021, sections 3.1 and 3.2].

14. For a look at Weyl's contribution to the debate on the foundations of mathematics, considering its different phases, see [Mancosu 2010, 259 *ff.*].

15. Despite the differences already pointed out (see footnote 9) between the Brouwerian and the phenomenological notion of intuition, and despite the fact that Weyl's shift to the intuitionist camp in the early 1920s coincides with a distancing from phenomenology, Weyl sees non-superficial links between intuitionism and Husserlian phenomenology, and even fails to distinguish between the specificities of the two. "In ways that are still not completely transparent", Weyl's interpretation of intuitionism "involved amalgamating phenomenological intuition, an 'originary giving' intentional act, with Brouwer's notion of 'primal intuition' " [Ryckman 2005, 114]. After moving in the mid-1920s from intuitionism to a position closer to Hilbert, he claims:

> If Hilbert's view prevails over intuitionism, as appears to be the case, then I see in this a decisive defeat of the philosophical attitude of pure phenomenology, which thus proves to be insufficient for the understanding of creative science even in the area of cognition that is most primal and most readily open to evidence—mathematics. [Weyl 1928] (See also [Marion 2004, 129 *f.*] and [Mancosu & Ryckman 2002, 148 *ff.*])

This association of the inadequacy of intuitionism with that of phenomenology seems to be due to Weyl's failure to recognize that the intuition of Husserlian phenomenology is not reducible to that of intuitionism. This conditions to a considerable extent Weyl's account of the relationship between Brouwer's intuitionism and phenomenology. Certainly Weyl, together with Husserl's student Oskar Becker, is one of the first authors to consider intuitionist mathematics from a phenomenological point of view [Van Atten, Van Dalen *et al.* 2002, 215], but it seems certain that there are considerable points of divergence; for example, mathematical entities are not timeless for Brouwer, whereas they are for Husserl [Marion 2004, 133 *f.*]. A perspective on the possibility of a phenomenological grounding of intuitionism in the epistemology of mathematics is given by Berghofer [2020].

continuum as a "medium of free becoming" [*Medium freien Werdens*] [Weyl 1921, 151, 153, 173].[16] Each element of the continuum must therefore be able to be defined by a *constructive* process of approximation that can be continued beyond any limit. The real number, then, is a number that can only be given in an indefinite approximation; therefore, the imprecision or approximation that is the essence of the intuitive continuum is found in the very concept of the real number [Weyl 1921, 151 *f.*], [Pradelle 2019, 183]. The continuum is thus a domain of *effectuation* where the real numbers fall, without being identified with an infinite non-countable set of elements [Weyl 1921, 153]. It is therefore clear to Weyl at this stage that Brouwerian intuitionism has come closer than any other approach to bridging the gap that exists between the intuitive and the mathematical continuum [see also Van Dalen 1995]. The rejection of the ideal of exactness and complete definiteness is demanded by the very nature of the intuitive continuum, which includes the *dynamic* character of the synthetic connection of its constituent elements, so that it will never be possible to construct this continuum from given elements that are set up from the outset as existing in their actuality and independent of one another. Therefore, it is not even possible to divide the continuum into fragments, precisely because it is not the union of disjointed parts.[17]

The set theoretical approach must then be replaced by a model that takes into account the *priority of the whole over its elements*, the *non-independence* of these elements, the *cohesion* of the elements in the continuum, and thus the *non-fragmentability* of the latter [see Weyl 1949, 41]. In other words, the formal analysis of the continuum should be based *not on the relationship of the element to the set, but on that of the part to the whole* [Weyl 1988, 5]:[18]

> The continuum falls under the notion of the "extensive whole," which Husserl characterizes as that "which permits a dismemberment of such a kind that the pieces are by their very nature of the same lowest species as is determined by the undivided whole".
> [Weyl 1949, 52]

This corresponds to the Brouwerian procedure, whereby the analysis by which the continuum is constructed splits an interval into parts or subintervals that are homogeneous to each other. The relation between part and whole, formulated here as the relation of inclusion of subintervals, thus replaces the relation between the set and its elements as the fundamental relation of the continuum [Van Atten, Van Dalen *et al.* 2002, 212].

What is claimed here is that, since we cannot intuit durationless points [Van Atten, Van Dalen *et al.* 2002, 211], we have to deal in our experience with a kind of *ontological primacy of the continuum over the discrete*. The

16. The translation is taken from [Sieroka 2019, 105].
17. In this respect there seems to be a strong criticism of Dedekind's method (see footnote 11) in [Weyl 1921, 173].
18. See also [Weyl 1921, 177]. This shift also takes place in Brouwer [Becker 1923, 426].

continuum can admit the discrete as its predicate (it can, for example, have discrete qualifications, such as the size and number of connected components), but the discrete cannot be continuum without ceasing to be discrete [Thom 1992, 138]. That the continuum can become discrete, and that from the discrete the continuum cannot emerge, is a statement that seems to capture a kind of eidetic necessity: from discrete sets of elements of dimension n cannot arise a continuity, which should be of dimension $n+1$. On the contrary, the definition of discrete elements of an arbitrary dimension n will only be possible within a given dimension $n+1$, through a process of "discretization", such as that of cutting a plane, a line, and so on. Thus, the only way to articulate the relationship of ontological foundation between the continuum and the discrete seems to be to understand the discrete as an "accident" of a sphere of a higher dimension in which it is grounded (e.g., points in a line, a line on a plane, etc.) and which represents a continuum with respect to the elements so identified in it [Thom 1992, 138]. Put in Aristotelian terms, continuity is analogous to the "matter" that unites species of the same genus. Species that are united by the same matter are species of a single genus, whereas species belonging to incommunicable genera cannot be added [Thom 1992, 139]. Husserl also refers to such relations in similar terms:

> Discontinuity as such refers to the lowest specific differences within one and the same next higher pure genus [...]. [Discontinuity] refers to the specifically differing moments only in so far as they are "*adjacently spread out*" over a continuously varying moment, namely the spatial or temporal one. [Husserl 1984, 250, my translation]

What distinguishes this "mereological" approach from that of set theory is that in it, unlike the latter, one does not start with already given basic objects (such as real numbers or transfinite sets with a certain topological structure), but these objects emerge only through constructive activity, by applying a scheme of division [*Teilung*] that can be repeated an infinite number of times [Weyl 1921, 178], [Baracco 2019, 599 *f.*]. Rather than being made up of points, the continuum should be thought of as being divisible into parts, and it is then necessary to operate on the basis of their inclusion relations. Therefore, there is no actual infinite set of points that exist in themselves, but only *a medium in which sequences of nested intervals can be freely generated* [Weyl 1921, § 4, 172], leading to a completely different understanding of real numbers.[19] Only this constructibility provides the mathematical expression

19. "By redefining primary elements of the construction of the real numbers as intervals rather than points, the mathematical continuum can finally be 'in harmony' with the intuitive one" [Kish Bar-On 2021, 54]. Since Weyl and Brouwer argue "that the intuitive basis of the concept 'real number' is to be found in choice sequences, it evidently follows that in Cantorian set theory we have an empty, purely conceptual (and possibly inconsistent) view of the continuum" [Van Atten, Van Dalen *et al.* 2002, 210].

for a significant property of the intuitive continuum, namely that each of its parts is infinitely divisible, or that it is "something that becomes by proceeding inwards without end" [*ein nach innen hinein ins Unendliche Werdendes*] [Weyl 1921, 172, my translation], [Pradelle 2019, 184 *f.*]. One cannot overlook the legacy of Brouwer, for whom the nature of the continuum is preserved on the condition that each point in it "never 'is' but always 'becomes' " [Van Atten, Van Dalen *et al.* 2002, 214]. The opposition is thus that of *being* versus *becoming* or *passivity* versus *activity*; continuity is associated by Weyl (at least in this period) with activity and becoming, and in a certain sense with "freedom". In contrast, Weyl associates discretion with the notion of something given, with passivity and constraint [Sieroka 2019, 105]. Instead of actual infinity, there is now the potential infinity of the infinitely repeatable division of the continuum, which is in some ways close to the Aristotelian position: the continuum does not consist of actual infinite indivisible parts, but is potentially infinitely divisible.

Thus, the points of a continuity, instead of being actually determined, are nothing more than the *ideal boundaries of the division process*, which can never determine a point with absolute precision, but can only achieve an infinitely increasing approximation. Sharp boundaries such as points, lines, surfaces, etc. are not ready-made objects, but objects *in becoming* [Becker 1923, 426]. The concept of the fully determined individual element in the continuum must then be regarded as a *limit idea*: it is nothing more than the representation of the end point of a process of division that ideally continues indefinitely [Weyl 1921, 177 *f.*]. (It should be noted, however, as Oskar Becker puts it, that the intuitive continuum is indeed given as a concrete entity, and that only the process of its mathematical rationalisation through the described process of division cannot be completed [Becker 1923, 432].) If the continuum itself is the only concrete thing, i.e., possessing an independent existence, then each element within it will be definable as a *limit* that cannot exist in isolation, but has an existence that is dependent on the concrete whole [Brentano 1933, 65 *f.*], [Smith 1995, 8], [Smith 1996, 295].

It is clear, then, that it is the very fluid character of the temporal continuum, as well as of spatial extension or movement, that cannot be captured by constructing continuity through the extrinsic union of pre-existing and well-determined elements [Weyl 1918, 69 *f.*], [Pradelle 2019, 182]; among other things, the paradoxes known to Zeno of Elea would be the result. It is not enough to put infinite indivisible points together in a finite segment to make continuity. What is required is the *relation of order* (which is *irreflexible* for every position, *connected* for every two positions, *transitive* for every three, and *asymmetrical* for all positions). It is the relation between the elements that constitutes the order of continuity; the elements alone cannot do this because they are external to each other, hence *discontinuous* [Corisch 1969, 537]. If the points as *relative positions* correspond to the elements of the line, then the order corresponds to the line itself as a geometric representation of temporal or spatial extension [1969, 537]. It is then this ordering relation that is infinitely divisible without requiring indivisible ultimate elements, and its whole is not the sum of the elements but the set of partial relations between them [1969, 538]. We can already see a circularity in Aristotle's discourse on the continuum: the continuum is presented as an already formed totality that gives meaning to its components. Weyl's present time is circular in the same way: each of its parts (past, present or future) has meaning only in relation to the others, and time itself is the simultaneous perception of past, present and future. The past is the present that is no longer here, the future is the present that is not yet here, and we understand this only when we are embedded in the whole of time or a segment of it [see Kerszberg 2019, 412]. The same applies to the continuity of the line, the plane, etc., which is not conceived as a series of basic pieces, but globally [Longo 1999, 406]. Precisely for this structural reason, for Weyl, points in time and space are nothing in themselves, and no exact determination of them is possible [Bell 2000, 266]. They are not genuine individuals and therefore cannot be characterised by their properties. In the physical world they are never absolutely defined, but only in terms of that coordinate system, which Weyl describes as "the inevitable residue of the eradication of the ego" in the world of the physical-geometric constructions [Weyl 1918, 72], [Livadas 2017, 104].

6 Mathematical continuum and time consciousness: A misguided approach

Although the notion of intuition that Weyl adopts in *Das Kontinuum* is not identical to the Husserlian one,[20] and the developments of his thought led him to adopt the intuitionist perspective, it is now time to show the phenomenological significance and fruitfulness of Weyl's approach to the problem of the intuitive continuum in his intuitionist period, regardless of whether he was faithful to the letter of Husserl's doctrine concerning continuum and intuition or not. Undoubtedly, the impossibility that Weyl sees in 1918 of a phenomenological grounding of the classical mathematical continuum on the intuition of phenomenal time, due to the gap between the two, leads him to a mathematical construction that goes far beyond a phenomenological description. This means that, at this stage, Weyl believes that a phenomenological theory of the continuum is not possible [Marion 2004, 140 *f.*]. However, the new approach adopted by Weyl after *Das Kontinuum* makes it possible to offer a rationalisation scheme for the continuum on the basis of its intuitive tenor, bridging the gap between the latter and its mathematical expression. In fact, as we have seen, the new

20. Again, even if the centrality of intuition in Weyl's epistemology is due to Husserl, his conception of intuition in the period under consideration is derived more from other sources, such as Brouwer and, particularly with regard to *Das Kontinuum*, Poincaré [da Silva 2017, 331, 345 *f.*].

mereological approach that Weyl adopts respects the non-objectifiable nature of the experienced continuum. In this way, it is finally possible to root the conceptualisation of the continuum in its intuitive dimension. This theoretical procedure takes into account the idealising process involved in the constitution of the ideal essence of a point, thus returning to a phenomenological motif present in Husserl. The ideal essence is subsequently articulated by an exact concept. The meaning of the latter can then be clarified on the basis of the phenomenological sphere of the intuitive continuum [Baracco 2019, 606].

Moreover, Weyl's analyses and some of Husserl's investigations concerning time consciousness seem to converge here—through the irreducibility of the dynamic-synthetic character of continuity—towards a model that seems to bring both close to a Bergsonian position (see [Feist 2002, 292], [Weyl 1918, 68 *f.*]). Indeed, for Weyl, since the days of *Das Kontinuum*, time consciousness paradigmatically represents the continuum that a mathematical theory must describe as faithfully as possible, and the conception of the temporal flowing continuity in this work is indeed strongly influenced by Husserl [Van Atten, Van Dalen *et al.* 2002, 203]. This may have something to do with the fact that Weyl was Husserl's student in Göttingen in 1904-1905, when Husserl gave his famous lectures on temporality [da Silva 1997, 277, 280].[21] Even if Weyl's intention is not to provide us with a phenomenology of the continuum, but rather to go beyond a purely phenomenological description of the continuum to offer a reconstruction of it in symbolic-mathematical terms, the phenomenological description is still the starting point [da Silva 1997, 279], and the symbolic reconstruction must be faithful to it, as we have seen.

> It is the phenomenon of the intuitive continuum that he feels he has not captured with the mathematical (or "arithmetic") theory of the continuum developed in *Das Kontinuum.* It is this phenomenon, in Weyl's eyes, to which Brouwer's development of intuitionistic real analysis does far more justice, and it is for just this reason that Weyl declares his allegiance to Brouwer in his famous paper on the "Grundlagenkrise".[22] [Van Atten, Van Dalen *et al.* 2002, 204]

21. However, the sources that Weyl mentions in the preface to *Das Kontinuum*, where he announces the theme of the relationship between intuitive experience and the formal concepts of mathematics, are the *Logical Investigations* [1984] and the first volume of *Ideas* [1976].

22. Brouwer's conception of the intuitive continuum (and with it Weyl's new intuitionist vision around 1921) also seems close to the Husserlian description of the experience of time [Brouwer 1907, 17], [Van Atten, Van Dalen *et al.* 2002, 205]. For a consideration, in relation to Brouwer, of the intentional structure of time consciousness described by Husserl [1966] in terms of a synthesis of multiplicity in unity, and with regard to the relationship between the empty horizons involved in time-consciousness and the incompleteness that intuitionism attributes to real numbers, see [Van Atten, Van Dalen *et al.* 2002, 206 *ff.*], where also a critique of the Cantorian approach is presented [2002, 209].

For our purposes, however, it is useful to point out that Husserl's long-term reflections on time consciousness are by no means uniform. It comprises at least three phases, roughly centred on three groups of lectures or manuscripts: the first phase corresponds to the famous lectures of 1905 plus other additional texts [Husserl 1966]; the second phase is that of the so-called *Bernauer Manuskripte* (1917-1918) [Husserl 2001]; the third phase comprises later manuscripts, written between 1929 and 1936 [Husserl 2006].

In the first phase, it is interesting to note that the structure Husserl assigns to temporality seems to be influenced by a somewhat atomistic representation. Of course, already here, the consciousness of time is not based on a mere succession of successive moments, but is determined by the *retention* of the content just passed, which brings the present of the actual "now" into a synthetic intentional dynamic. Nevertheless, here Husserl seems to recognise in the *impressional* present phase a kind of "originality" and autonomy with respect to the intentionality of retention, which comes only secondarily to modify it. In this way, time, although structured according to the dynamics of impression-retention, seems to emerge from a succession of instants that represent, as it were, the punctual elements through which the purely sensory, non-intentional content of the original impression gains access to consciousness in order to be modified by retention and thus become intentional. In this way, the sensible, *hyletic* moment is conceived as *de jure* independent of any intentional mediation in which it is *de facto* involved, which makes it problematic to clarify the dynamic flow of phases that should take place precisely through such mediation [see Schnell 2007, 187 *f.*].

There is no textual evidence to draw a parallel between Weyl's conception of time and Husserl's lectures; Weyl's phenomenological description of the constitution of time, which is contained in § 6 of the second chapter of *Das Kontinuum*, is essentially based on Husserl's analyses in *Ideen I* (§§ 81–82) [da Silva 1997, 281, 295], which do not reach the descriptive depth of the 1905 lectures and related texts, but which nevertheless report some of the basic assumptions present there. In his reference to these analyses, Weyl is of course unaware of the latent problems involved in their development as set out in the 1905 lectures, so much so that he regards them as prototypical of the flowing intuitive continuum in which no points can be fixed. Now, Weyl's attempt in the 1918 book to construct mathematical analysis by reflecting the structure of the intuitive continuum requires that real numbers must serve as the symbolic counterpart of temporal instants. From a systematic point of view, it may be of no little importance to note that the kind of temporal structure required here by the construction of mathematical analysis (if it is to be based on the intuition of the continuum) appears to be quite analogous to the model described by those early Husserlian investigations. It is precisely because this model does not correspond to the actual structure of immanent temporality, in which present instants have no autonomous existence, that Weyl's 1918 attempt was doomed to failure [da Silva 1997, 290 *f.*].

> If it is this intuitive continuum that analysis has to model then, concludes Weyl, classical analysis is unacceptable. The very notion of a point in the continuum lacks intuitive support. So, if real numbers are seen as formal counterparts of instants, they cannot be given independent existence. [da Silva 1997, 295]

7 Mathematical continuum and time consciousness: A more promising approach

By entering into a decisive new phase of his reflection, Husserl finds a way to overcome the ambiguities of his first approach to the phenomenological description of the constitution of time. In fact, in the *Bernauer Manuskripte* of 1917-1918 (a detailed and comprehensive discussion of which is, of course, not possible here), a radical change takes place, which ultimately corresponds to what Weyl mistakenly thought he had always seen in Husserl's reflection on time. Here Husserl no longer speaks of original impressions, but of something like a *maximal core*, and this core is such that it is originally included in the intentional dynamic [Husserl 2001, 32], [Schnell 2007, 184]. The original time process is entirely intentional and has no unconscious element [see Husserl 2001, 42]; here there is no such thing as a primary impression, as an individual, atomic and primitive element of the continuous flow of time. There is then no longer a series of original "now-points", but a double continuum of intentions, a retentional and a *protentional* graduality, the intersection of which only constitutes the consciousness of an original presence through the identity between the maximum point of positive graduality and the minimum point of negative graduality.[23] This now leads to the impossibility of the independence of the hyletic moment in its pure presence [Husserl 2001, 161], [Schnell 2007, 194]. Even in Husserl's analysis of time, there is no primary individual element, but only the synthetic whole of the dynamic time continuum, in which the present individual element is only an ideal boundary. Each point in the continuum thus appears as a *non-independent moment* of an intentional complexion to which does not correspond the real, determined and transparent presence of an individual out of which the temporal continuum would be constituted. This non-independence is to be understood in the sense clarified by Husserl in the 3rd *Logical Investigation*, as the non-self-sufficiency of an abstract moment of a whole, which requires a *foundation* by other moments. In this case, the abstract character of each point of the time continuum means that it cannot be thought of as existing independently, but only as a non-independent moment of the continuous medium that constitutes the concrete founding whole.

23. For this discussion see [Schnell 2007, 189].

The fact that the improvements made here seem to echo Weyl's approach in his 1921 "Über die neue Grundlagenkrise der Mathematik", discussed above, despite the fact that not only could Weyl not have known the *Bernauer Manuskripte*, but that he was at that time more influenced by Brouwer than by Husserl, is perhaps indicative of a kind of structural necessity inherent in the subject itself, which goes beyond philological-historical contingencies. In other words, while it is certainly true that Weyl's new approach to the problem of the continuum seems to be committed to a model of temporal continuity similar to that described by Husserl, this is true exclusively with regard to Husserl's revised account of the constitution of time. It is only in this latter account that it is possible to understand how the intuitionist procedure "of modelling the intuitive continuum mathematically as the species of the more or less freely proceeding convergent infinite sequences of rational intervals" [Van Atten, Van Dalen *et al.* 2002, 209] can be applied to a Husserlian time continuum. As Weyl (and Brouwer) realised, we do not "experience a durationless now-point in our awareness of the flow of internal time"; consequently, if we want to mathematically model such intuited time in a faithful way, without violating its fluid, non-decomposable dynamic structure, all we can do is try, in the intuitionist way, "to approach a durationless point in this intuitive continuum by an infinite sequence of nested rational intervals the lengths of which converge to 0" [Van Atten, Van Dalen *et al.* 2002, 209]. The durationless point thus obtained would then be the real number, which here too, as for Husserl's analyses of time in the *Bernauer Manuskripte*, is not conceived as a discrete element that would be independent of the procedure by which it is obtained, which cannot but have its starting point in the intuitive representation of the continuum.[24]

8 From experience to idealisation

In *Das Kontinuum* Weyl argued that a point such as a real number is not something accessible to intuitive experience, but can only be grasped through thinking and idealisation. This was the reason why Weyl believed that classical analysis was incapable of giving an intuitive foundation to the real number, because it relies on the positing of durationless points, which can never be given in intuition. It is the apport of Brouwer's new intuitionist approach to provide an intuitive foundation for the real number in the way roughly described [Van Atten, Van Dalen *et al.* 2002, 209 *f.*]. As far as Weyl is concerned, it is clear how the relationship between the essentially inexact morphological-intuitive sphere and the exact mathematical-formal sphere is to be articulated: instead of reducing the continuous nature of spatio-temporal experience to the

24. It should be noted, however, that in the intuitionist procedure the point corresponding to the real number is the sequence of nested intervals itself, and that each interval is a part of it [Van Atten, Van Dalen *et al.* 2002, 212].

analytic-formal identity of its constituent elements, Weyl recognises that, on the contrary, the formalisation of a continuity is only possible "afterwards", through the imposition of a scheme that idealises the flowing continuity by giving it exactness and absolute determination. Such a scheme is provided by the concept of the real number and that of the continuous function, which allow for an analytic theory of the continuum, as he already states in his early period [Weyl 1918, 71]. The dimensionless point is therefore a derived conceptual construction, it is a posterior reconstruction of set theory, which assembles the points *ex post* to reconstruct the line. It is this reconstruction that gives rise to the inversion of set theory: the continuum as a set of points [Longo 1999, 404], which, however, leaves intact the fact that mathematical, exact and objective representation does not replace the phenomenological structures given in experience to capture the truth in itself of things beyond the way they appear, but is based on these structures and therefore presupposes them. Ideal elements beyond what is given in the intuition of space and time are thus correlates of an idealisation process that focuses on the "*limits* of series of possible intuitions" [da Silva 2017, 333].

After this discussion, however, it must be kept in mind that Weyl does not arrive at a definitive solution to the problem of the continuum and its mathematical expression, but continues to work on it, so that even in a text written in 1954, one year before his death, the issue is still regarded as a "serious affair" [Weyl 1985], [Kish Bar-On 2021, 54].

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