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THE EASIEST AND THE MOST DETERMINED: FINAL CAUSATION IN LEIBNIZ'S OPTICS

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Since 1678 at the latest, and onward throughout his whole philosophical itinerary, Leibniz will claim that final causes should be considered of the highest importance both in philosophy and science. In arguing in defence of their role, Leibniz will repeatedly claim, as a direct response to Descartes statement on final causes being useless in natural philosophy, that final causes can have a heuristic role, for they help in finding natural laws, and that final causes can be the basis of a potentially complete model of explanation of phenomena. Anything in nature can therefore be explained in two ways, by means of efficient causes and by means of final causes[1]. Notwithstanding the generality of this claim, Leibniz's examples on how final causes can provide an explanation of natural phenomena will be limited to two subjects: biology and optics. Among these two, optics is the only field where final causes are related to the demonstration and discovery of laws of nature. In this paper, I will therefore concentrate on final causation in optics[2] and, albeit something will be said on their heuristic role, on how they provide a method for deriving natural laws. I aim at showing that the symmetry existing among the two models at the level of explanation of phenomena is not mirrored

1. "Omnia naturae phaenomena explicari possent per solas causas finales, perinde ac si nulla esset causa efficiens; et omnia naturae phaenomena explicari possent per solas causas efficientes, quasi nulla esset finalis", *Definitiones cogiationesque metaphysicae*, A VI 4, p. 1403 (see: Bibliography for abbreviations).
2. An introduction to Leibniz's optics will soon be available, in J. Mc Donough, "Leibniz and Optics", in M. R. Antognazza (éd.), *The Oxford handbook of Leibniz*, forthcoming.

at the epistemological level. Differences on the epistemological status change through the years: while in earlier years the uses of teleological proofs in optics are more problematic then proofs by efficient causes, in later years they acquire a stronger epistemological status than the latter. I hope to show that this evolution, as well as some changes in the metaphysical background, can be understood when related to specific optical problems, mainly refraction on convex surfaces which drives the evolution from the principle of the easiest path to the principle of the most determined, which is a redefinition of a concept first employed by the Jesuit André Tacquet.

The Structure of Leibniz's Demonstrations and its Metaphysical Background: Some Preliminary Remarks

Notwithstanding their evolution, all the demonstrations of optical laws by means of final causes provided by Leibniz share the general structure of his explicitly mentioned predecessors: Heliodorus of Larissa, Hero, Ptolemy[3] and Witelo in the case of reflection and Fermat who extended the method to refraction. It consists in considering two arbitrarily chosen starting and ending points for the optical path and then seeking one point on the mirror (reflection) or on the ideal line or surface of separation of two different media (refraction) that univocally determine the optical path. In this scheme, optical laws are obtained by considering a property (in most cases the minimization of a defined quantity) that pertains to only one path to then derive another geometrical relation implied by the chosen path which is the desired law. This structure had been subjected to many criticisms, most of them raised in the context of the quarrel that opposed Fermat, first to Descartes and later to De La Chambre and Clerselier on the issue of refraction. While Descartes aimed mainly at defending his own demonstration, Clerselier made the criticism to Fermat's principle explicit. A first point raised by Clerselier is that this type of demonstrations assumes intentionality in natural

3. Ptolemy's commitment to demonstration by minimum is problematic. The main source is probably the attribution of a writing on Catoptrics which is now considered Hero's work. While some further research on the issue is needed, it is hard to tell whether Leibniz relies on this attribution or if he had other sources. On Hero and Ptolemy demonstrations see A. Smith, *Ptolemy and the Foundations of Ancient Mathematical Optics*, American philosophical society, Philadelphia 1999, p. 79-83; see also the remark on the edition of Ptolemy's optics: C. Ptolemaus, *L'optique de Claude Ptolémée: dans la version latine d'après l'arabe de l'émir Eugène de Sicile*, éd. A. Lejeune, Leiden, Brill, 1989, p. 98.

bodies. The second main objection is that optical laws should be proved by means of causal demonstrations, while minimum principles cannot provide causes, because they do not determine anything in nature. Only forces make bodies act. Coupling these two points, Clerselier criticism might be summarized as follows: demonstrations by minimum treat their *desideratum*, a point on a surface, as if it was chosen by light rays, not acknowledging that it is fully determined by the nature of the incident ray.[4]

In a text from 1677, Leibniz underlines another problematic point of ancient demonstration by minimum: "Et paralogismus in eo [Hero and Ptolemy demonstration of reflection] erat, quod improprie dicimus, ex omnibus nisus eligi commodisssimum sortiri effectum; iam ipse nisus est aliquis effectus"[5]. In Leibniz's view, the way in which this demonstration has been developed arbitrarily considers as a sort of privileged effect the change in direction, not considering the continuity of the causal process. It is a strange sort of remark, to some extent, because the minimum strategy may apply to rectilinear transmission too, therefore acknowledging for each "state" of the *nisus* as an effect. However, it shows that for Leibniz, final causes should not be referred to a specific effect produced, but to properties characterizing the relation of order given by the consideration of whole path. In his own treatment of the issue, Leibniz tries to find a way to assume the validity of Clerselier remarks and yet dismisses his conclusion, the rejection of demonstrations by minimum[6]. In Leibniz's view, then, the method by final causes does not supply to a lack of determination of efficient causes, which are in principle enough to demonstrate the law, as the thesis of the two complete and autonomous methods of explanation grants. It simply looks at phenomena from another (to some extent more fundamental)

4. Clerselier to Fermat (may the 6th, 1662), in R. Descartes, *Lettres…*, vol. III, éd. Claudie Clerselier, Paris, chez Charles Angot, 1667, p. 277 and ff.
5. LNS, p. 71. Gerland reading is problematic: "aliquis" should probably be substituted either by " aliqui" or by "alicuius".
6. "Huic argumentationi recentiores quidam objicere solent, radium ex *A* egredientem cognitione praeditum non esse nec quaerere an ad *C* iturus sit, et quanam optima via eo sit perventurus, sed caeco impetu incurrere in superficiei refringentis punctum, ad quod concepta jam directione feretur; atque inde secundum Mechanicas leges repercuti. Sed hi non cogitant argumentum veterum esse sumtum a causa finali, et non radium quidem, sed naturam tamen legum opticarum fundatricem cognitione praeditam esse, quidque optimum et commodissimum futurum sit, praevidere", *Definitiones cogiationesque metaphysicae*, A VI 4, p. 1404.

point of view: not how things are realized but why they are in this way. Therefore Leibniz can underline the teleological structure of this argument, explicitly denying intentionality in bodies and ground their behavior in God's ends.

This explicit reference is one of the important things Leibniz believes he added to Fermat's demonstrations[7]. Fermat had limited himself to the statement that minimum principles provide a reduction of optical problems to purely geometrical ones; Leibniz provides a metaphysical grounding: minimum principles work in optics because they express God's ends and therefore they have been the object of His choice in creating the world. The method of final causes is then a comparison of possible order and he is the first, among his aforementioned predecessor, to explicitly rely on this interpretation. As the above short description of the structure of demonstrations shows, Leibniz does not directly compare different laws, but rather physical possible answer to a given problem[8]. If this implies a comparison of laws, it is because, in Leibniz's eyes, each physical possibility can be seen as a consequence of some possible law, because each of them, no matter how complicated, admits a law-like description.

A grounding of the passage from physical possibilities to possible laws can be found in the more general, and therefore not explicitly referred to this problem, consideration on the impossibility of absolute disorder in paragraph VI of the *Discours de Metaphysique:*

> […] non seulement rien n'arrive dans le monde, qui soit absolument irregulier, mais on ne sçauroit mêmes rien feindre de tel. Car supposons par exemple que quelcun fasse quantité de points sur le papier à tout hazard […]. Je dis qu'il est possible de trouver une ligne geometrique dont la notion soit constante et uniforme suivant une certaine regle; en sorte que cette ligne passe par tous ces points, et dans le même ordre que la main les avoit marqués. Et si quelcun traçoit tout d'une suite une ligne qui seroit tantost droite, tantost cercle, tantost d'une autre nature, il est possible de trouver une notion ou regle ou equation commune à tous les points de cette ligne, en vertu de laquelle ces mêmes changemens doivent arriver[9].

7. "[…] mais bien loin de dissimuler que ce principe a quelque chose de la cause finale, comme on avoit objecté à M. Fermat […] je l'en trouve plus bel et plus considerable […]". Cf. *Tentamen Anagogicum*, GP VII, p. 273.
8. Already in 1677, when, as said, doubts are raised on the demonstration by minimum, Leibniz defines the set of possible geometrical descriptions as "aggregatum omnium possibilium.", LNS, p. 71.
9. G. W. Leibniz, *Discours de métaphyisique*, A VI 4, p. 1537-1538.

From the point of view of the use of teleology in nature this passage is a sort of logical condition for stating that the method of final causes implies a comparison of types of order, at least establishing the existence, for every point we choose (e.g. on mirrors), of a mathematical description for the resulting relation. The method of final causes implicitly compares possible orders, while the relation that allows their comparison, the equation that defines the set of physical possible answer to the problem, is a sort of meta-description of possible orders that may rule a phenomenon. To put it otherwise, the abstract possibility of determining a set of laws for randomly produced geometrical points or entities grants that for every phenomenon that allows geometrical description each possible case will allow a law-like description, no matter what types of physical constraints we need to add to define the set of possible cases. Another more general requisite for this method is obviously the existence of various possible paths that light could follow. It requires, and uses, what can be labeled as "contingency of the path", its possibility to be otherwise (even if reaching the very same point). Strictly speaking, the demonstrations rely only on the possibility of different geometrical solutions to a given problem. In the frame of Leibniz doctrine of contingency where non-necessary things are contingent and still possible if their notion does not imply a contradiction, by the existence of different geometrical solution to a problem one can move to the possible existence of physical solutions. If what the method by final causes seeks is a law, the existence of law-like description and of possible physical realization of the various geometrical alternatives, can be coupled in the claim that contingency, both of paths and laws, is a condition for applying the method of final causes[10].

The Easiest Path and the Epistemological Status of the Method by Final Causes

To have this scheme working, a definition of the characteristic that allows to derive the law is needed. During the late seventies and in the *Unicum opticae catoptricae dioptricae principium*, Leibniz labels it the easiest path,

10. On the background, there is obviously the more general problem of the geometrization of physical entities, which cannot be addressed here. While this regards any use of geometry in physics, contingency has a more specific role, connected mainly to the demonstration by final causes. On the connection between optics and contingency, with a slightly different interpretation, see J. Mc Donough, "Leibniz's Optics and Contingency in Nature", in *Perspectives on Science*, 18, 2010, p. 432-455.

when using it in the physical sense. In ancient demonstration, concerning only reflection, the property of the path by which the law is derived was identified with the shortest path. Since Fermat extension of this method of demonstration to the case of refraction this was no longer enough, for it obviously does not work in this case. Notoriously, Fermat, recalling Galileo's work on free fall, had proposed that the quantity to minimize should be time: "natura operari per modos et viam faciliores et expeditiores"[11].

One of the first definition of the easiest path given by Leibniz comes from an untitled text dated 1677: "Natura ergo suspensa nisus suos non alio exercet modo, quam quo minimum immutatur, seu quam minimum amittitur virium, id est facillimo"[12].

How this definition of the easiest should be implied in geometrical demonstrations remains puzzling, for neither a measure of the force, nor a scheme for unifying the description of its variation in the various possible cases is given. In almost contemporary text, there is already a definition that allows a direct employment in geometrical optics: the easiest path is given by the composition of the distance and of the resistance of the medium. Easiest has then different meaning than in Fermat and in doing so Leibniz suggests to directly employ a physical property of the environment in which the phenomena appear rather than consider only its effect on the ray of light, as in Fermat or in his own previous definition by minimum change. One may nonetheless wonder what the relevance of this change was in Leibniz's eyes. As I read it, the main reason is epistemological. Even before precisely defining the easiest path, Leibniz had claimed that there is something valuable in the reasoning by minimum even admitting instantaneous activity of light: "Videtur pro Ptolomaei et Fermatii ratiocinatione dici posse aliquid non comtemnendum nimirum ponendo cum Aristotele lumen operari in instanti quod etiam admittit Cartesius"[13]. As it is well known, the debate on whether the light acts instantaneously or not was more than alive, even after Rømer first measure of light speed in 1676: the definition of "easiest path" given by Leibniz makes the use of final causes independent of this problem. Generalizing, the definition of the best choice in given cases should be used by taking into account which variable cannot be eliminated; in the present case, the fact that

11. P. Fermat, *Methodus ad disquirendam maximam et minimama, Synthesis ad refractiones*, in Id., *Oeuvres de Fermat* vol. IIII, éd. C. Henry and P. Tannery, Paris, Gauthier Villars et fils, 1896, p. 173.
12. LNS, p. 71.
13. LNS, p. 59.

light travels some distance and behaves differently in different media. In eliminating time Leibniz has made his demonstrations independent of any hypothesis on the nature of lights. The relevant variables should be derived by considering what cannot lack in every description of the phenomena rather then what depends on hypothesis: the condition of applicability of the method by final causes should be analytically derived by the description of phenomena.

This is at least one of the main basis of the heuristic function of teleological demonstrations: laws ruling some phenomena can be derived even not knowing exactly what that thing is and how it works, as the *Unicum* makes clear: "[…] praeter admiratione divinae sapientiae pulcherrimum nobis principium praebant inveniendi earum quoque rerum proprietates, quarum interior natura nondum tam clare nobis cognita est […]"[14].

As relevant as the heuristic role is for Leibniz, final causes have a more fundamental role in optics: given that the principle of the easiest path can be applied to rectilinear transmission, the geometrization of optics by means of a principle derived by final causes is obtained independently by consideration on the nature of light: "Reduximus ergo omnes radiorum leges experientia comprobatas ad puram geometriam et calculum, unico adhibito principio, sumto a causa finali […]"[15].

In later text, the independence from hypothesis is used to give to demonstrations by final causes a higher epistemological status compared to the ones by efficient causes, since they are absolute, while the latter relies on some hypothesis. This stronger status emerges clearly in a marginal note on a letter by Gackenholz written in November 1694. Where Gackenholz praises the demonstrations published in the *Unicum*, Leibniz writes: "Causas non tantum finales, sed et efficientes dare possum. Verum hoc interest quod illae sunt absolutae, hae hypothesi nituntur"[16].

14. G.W. Leibniz, "Unicum opticae, catoptricae et dioptricae principium" in *Acta Eruditorum*, Mensis Junii 1682, p. 186; the same idea appears in the controversy with Stahl: "Nondum hactenus pro certo affirmare possumus, ita exacte nobis cognitam esse naturam radiorum lucis, ut ex causis efficientibus reddere ratione possumus legum, quas radii in reflexione refrationeque observant […]. Sed adhibita causa finali mira facilitate prodeunt leges, quas experientia comprobat.", in G. W. Leibniz, *Obiezioni alla teoria medica di Georg Ernst Stahl*, ed. A. Nunziante, Macerata, Quodlibet, 2011, p. 34.
15. G. W. Leibniz, "Unicum opticae…", cit., p. 186.
16. Marginal note on a letter from Gackenholz (A III 6, p. 231). Referring to Gackenholz suggestion of extending this method to the problem of impact Leibniz adds a short, but

If what said so far is correct, this statement should be distinguished by another leibnizian thesis that partially undermines the epistemological status of the efficient causation model, namely the fact mechanical models cannot ground their own principles, which depend on final causes. Both these are related to God's choice of laws of nature, but the one at stake here regards more the status of the demonstrations then the grounding of principle, the possibility to derive all the needed assumption, more than their proofs. From this point of view, teleology in optics has a different role than architectonical principles, such as continuity and equivalence of cause and effect, mainly because it defines specific structures of demonstrations and the grounding of its admissibility in natural philosophy. Notwithstanding the employment of principles, optical teleology is here a model of explanation, more than a principle to which natural laws should cohere[17].

This status of demonstrations by final causes faces two different problems. First, the absence of hypothesis seems hard to reconcile with the fact that Leibniz uses some assumptions on the nature of medium to account for refraction, which define in each concrete case the direction of the refracted ray with respect to the perpendicular at the (tangent of) the ideal line of separation of the two media. Once again, the nature of the media was one of the central issues in the debate among Fermat and Descartes. In short, while Fermat had supposed that light travels faster in less resistance media, Descartes thought that light acts more easily in more resistant media which in his view are to be described in terms of hardness and softness. These diverging accounts lead to two different interpretations of the constant ratio among the sinus of incidence and of refraction: for Fermat, the two angles are directly proportional to velocities, while for Descartes, they are inversely proportional to resistance[18].

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very interestingt remark: "Non debet applicari ad repercussionem corporum solidorum" (*ibidem*). A strange note, indeed, given the repeated claims on the universality of such method. It seems that, at least at this time, Leibniz had not, nor did he seek, a definition for the meta-description of possible laws in the case of impact such as the one provided by Maupertuis's principle of least action.

17. On the role of architectonical principles see F. Duchesneau, *Leibniz et la Méthode de la science,* Paris, Presses universitaires de France, 1993, chap. IV.

18. On the debate among Fermat and Descartes see A. Sabra, *Theories of light*, Cambridge, Cambridge University press, 1981, p. 93 and sq. As Sabra had rightly pointed out (ibid, p. 138), different interpretation on the media makes the assumption that the debate regarded the *same law,* as Fermat and Leibniz seem to believe, problematic.

In the *Unicum opticae*, Leibniz accepts Descartes view on the constant and his general claim that the easiest action occurs in more resistant media. For Leibniz, the problem in Descartes account relies in his view on the nature of media[19]. More precisely, by defining resistance in terms of hardness and softness and grounding his reasoning in the analogy with motion, Descartes was unable to explain why light coming going from water to air and then back to water recovers his strength and direction. In other words, in Leibniz's eyes, Descartes' account is unable to explain the reversibility of optical path. Leibniz account of the media has this scope, among other. The central point of his account is to describe resistance in terms of rareness and density and connect them with the concept of illuminability. The more the medium is illuminable, the higher numbers of parts are acted upon by light, therefore the single ray is weaker, while in a lesser illuminable media, that resists more to the action of light each ray is stronger:

> Pro refractione explicanda considerandum est medium magis resistens lumini (sine opacitate tamen) illud esse videri quod magis impedit lumini diffusionem seu distributionem per plures medium partes, idque dici potest minus illuminabile, lumen enim natura se diffundere nititur […] verum constat ex Mechanicis principis eundem ictus pluribus corporum simul impressum minorem singuli vim dare, quam si omni eorum fuisset inflictus ideo fiet ut in medio magis resistens diffusione luminis, seu secundum pauciores partes affecto, singulae partes tanto fortius afficiantur, in magi illuminabili plures sed debilius[20].

The result is that when the intensive action of light is stronger, the extensive is weaker and vice versa[21]. In earlier texts this idea was related to the difference between *conatus simplex* and *conatus* or *nisus continue reparato*[22]. However, this reference to light as a continuously repaired *nisus* is not explicit in the *Unicum*. Here, the basis for the account on the media needed for Leibniz's proof is analytically derived from the differences in illuminability, which in turn are connected only to the number

19. On the issue see S. Bachelard, "Maupertuis et le principe de la moindre action", in *Thales*, 9, 1958, p. 3-36.
20. G. W. Leibniz, "Unicum opticae…", cit., p. 189-190.
21. Cf. LNS, p. 72.
22. "[…] Omnis motus a vi continuo supplemento reparata refringetur ad perpendicularem, quia omnia vis continuo supplemento reparata fortius agit in resistens quam cedens […] Non ergo de Lumine tantum sed in generale omni nisu continuato verum est, eum esse fortiorem in resistem quam statim cedens", LNS, p. 66.

of parts and not to their nature, making the non-hypothetical character of demonstration by final causes coherent with Leibniz's account.

In the later *Tentamen Anagogicum*, Leibniz seems to be much more open to the idea of taking time as a relevant variable, for in two occasions, one referred only to rectilinear surfaces the other more general, he explicitly mentions it and refers to the direct proportionality of resistance of the media. Nonetheless, even in this text, the demonstration of the law of refraction is given by considering (and geometrically interpreting) only resistance and the constant ratio of sinus is still given by inverse proportionality, as in Descartes' account. It is hard to tell whether Leibniz was simply trying to show how (a renewed version of) Fermat's principle could be seen as a consequence of his approach or if he had come to consider light diffusion as a non-instantaneous process. It remains that to prove the law of refraction, Leibniz considers only resistance and distance as he did in the *Unicum*. As we shall see in the following paragraph the *Tentamen* presents nonetheless a new scheme of teleological demonstration, in which a more general answer to the question of the absolute character of the method by final causes emerges.

Reflection on Curved Surfaces: a Counterexample to Minimum Principles

The second problem faced by demonstration by minimum is even more relevant and is, in my opinion, the guiding problem whose solutions drive the evolution of Leibniz's accounts on the issue: from the earliest doubt and restriction in the Seventies, passing through the partial solution adopted in *Unicum principium* to a full reformulation of the proof given in the *Tentamen Anagogicum*, the problem is the following: in the case of reflection on concave surfaces demonstrations by minimum face an evident counterexample. Given a semi-circular mirror and having the two extremes of the diameter as starting and ending point, among all the paths that touch the mirror in just one point, the path in accordance with the law of refraction, the one that cuts the half-circle in two equal cords, is the longest possible. Since his earlier mention of demonstration by minimum, Leibniz is aware of the problem, and in its first appearance it seems a reason to dismiss the demonstration:

> Ptolemaei ratio de minima in speculis concavis non succedit, si subintelligatur Tangens, non tamen res succedere videtur, nec forte remedium meum valet, de actione instantanea quod Fermatianae opinioni adhibui[23].

The *Unicum* solution is a reinterpretation of an idea that appeared in a text from the late seventies. In the case of curved surfaces one should consider the tangent plane, which defines the simplest case in which the principle works. In its first appearance, this consideration leads us to differences in the level of certainty of reasoning by final cause. It is a demonstration only in simplest cases and when we know for sure they are simplest. The reason is that in simplest cases what is the best (*perfectissimum*) can be determined. Otherwise it is just a conjecture, based on the idea that complex cases are composed of the simplest, so that the rule of the best can be seen as a sort of sum of perfection realized by simplest cases. When the conjecture saves phenomena, as in optics, it can be labelled a hypothesis. Given this problematic status, there is an even stronger limitation; final causes in physics should be admitted *only* as heuristic tools, while accurate demonstration comes from consideration on how light operates[24].

This sort of restriction will disappear in the *Unicum*, in which the easiest path acquires the status of the fundamental principle to derive laws and grounds the geometrization of optics. Nonetheless it programmatically considers only simple cases. Not only proofs for laws of geometrical optics are given only for rectilinear surfaces; in the case of curved surfaces, the *via omnium facillima* has to be determined by means of the tangent to the surface, therefore reducing complex cases to the simplest[25], because it is by means of the tangent, that the angles that enters the optical law can be determined. This solution may be seen as problematic at least in some of its metaphysical consequences, since

23. G.W. Leibniz, *Ratio aequalitatis angulorum reflexionis et incidentiae* (A VIII 1, p. 186).
24. "Si vero radio incidant in superficiem curvam […], concavam vel convexam, consideranda non est ipsa siperficies, sed plano eam tangens in puncto incidentiae. […] Haec ratiocinatio petita est a causa finali seu consilio dei in natura perfectissime operanti, et vim demonstrationis haberet in rebus simplicissimis; ubi determinare potest quod sit perfectissimum. Verum a magis compositis, vel etiam in his ub incertum est composita an simplicia sint vim tantum habet conjectura nam in compositis disceditur a semplicitate vel ideo, quia constantia natura esigit, ut regulae perfectionis a simplicibus sumantur. […] Verum hoc loco plus aliquid quam conjectura est, transit enim in hypothesin, quia per eam phenomena accurate et feliciter salvantur. […] Itaque Methodus a causa finali minima in physica recipienda est […] servit enim ad inventionem", LNS, p. 62.
25. See G. W. Leibniz, "Unicum opticae…", cit., p. 185.

it implies that the principles is not "physically" realized, so to speak. While it is by means of the tangent that the angles are determined, it is also true that in the complex case, the *aggregatum omnium possibilium*, the possible paths and therefore the possible laws, are defined by the curved surface and not by the tangent; or, if one prefers they are defined by the whole set of tangents to the curve. Therefore, while by means of reduction to the simple case, the principle might be maintained, it cannot be employed in a proof for the law in complex cases, because the law itself is needed to determine which reduction to rectilinear surfaces, among the ones given by the whole set of tangents to the curved surface, should be considered. From this point of view, the reduction to simple cases, when taken in a strict sense so to grant the logical status of the principle, makes the easiest path strategy unable to ground demonstration for complex cases even when the minimum strategy would succeed with variation defined by the actual surface, as it happens in reflection on convex surfaces.

In the *Tentamen Anagogicum*, Leibniz tries to redefine optimality to include the complex cases. The easiest path does not disappear, but the driving concept is here that of the most determined path which was already introduced, but not defined, in the *Discourse de Metaphysique*.

A Source For the Concept of "Determined": André Tacquet's *Catoptrica*

The concept of "determined path" (*via determinata*) in optics, in connection with demonstrations by minimum, had previously been used by André Tacquet (1612-1660). Tacquet was the one who brought to attention the problem caused by refraction on concave surface in demonstration by minimum in his *Catoptrica*[26], published posthumously in 1669. To my knowledge, he is the first that makes an explicit reference to the fact that in concave surfaces it is quite easy to build a case where the path travelled by light (or by species) is the longest

26. André Tacquet, *Catoptrica,* in Id., *Opera Mathematica R. P. Andreae Tacquet antuerpeiensis e societate Jesu…,* éd. Simone Laurentio Veterani, Antwerp, apud Jacobium Meursium, 1669, p 213-264. Leibniz sometimes mentions Tacquet mathematical works (in A VII 1, p. 178, on the inscription of maximal circles in spherical triangles; in A VII 2, p. 803, as a great contemporary geometer together; see also A VII 5, p. 297 and 557) though not, to my knowledge, his optical writings, which nonetheless where contained in the mentioned edition, available to Leibniz.

possible, relying both on empirical confirmation and on the theorem demonstrated by Serenus (300-360)[27].

In proposition III of the first book of his *Catoptrica*, Tacquet introduces what he calls the *fundamentum* of the whole catoptrics, i.e. the law of refraction. He then adds that the real cause of this equality had not been found yet and discusses previous demonstrations. While focussing on the *ratio Vitellionis* he observes that since there is an evident counterexample, the principle "naturam viam brevissimam affectat" cannot identify the *ratio aequalitas anguli*. Still, Tacquet maintains the validity of the shortest path principle, which he considers an axiom. Strictly speaking, the main reason for maintaining the validity of the *via brevissima* axiom is that nature follows it whenever possible. Implicitly, Tacquet is redefining his own axiom as having an if-then structure (If there is a shortest path, then nature will follow it): cases of non-existence of minimum do not count as counterexample. Coherently, in Proposition 8 which makes the consequences of Serenus' theorem explicit, there is a sketch of a proof that on semicircular mirror there is no shortest path. However, it seems that, beyond the existential restriction, Tacquet is also suggesting that the validity of the principle will somehow be granted by the assumption that when the path is not a minimum it still is the only determined one, i.e. a maximum. By combining both these suggestions, even if the minimum strategy is unable to provide a reason for refraction, Tacquet can maintain a sort of privileged status for the shortest path axiom, also testified by the fact that the proof that in rectilinear surfaces the equality of angles is consistent with the axiom, is the only proposition, which is labelled also as a theorem.[28] A theorem never used in the ongoing of Tacquet's *Catoptrica.* Tacquet's attempt to reconsider the use of minimum strategy in the light of the problem posed by refraction on curved surfaces has then led him to assign it a preeminent status and yet to make it almost useless in optics. On the one hand, somehow paradoxically, the very fact that Tacquet considers the proposition on *via brevissima* as an axiom seems to prevent him to give it any grounding which could allow its use in a causal demonstration, as Leibniz's theory of divine choice does. On

27. In the edition available to Tacquet (Serenus, *De Sectione coni*, in *Apollonii Pergaei conicorum libri quattuor cum…*, éd. Federico Commandino, Bononiae, 1566), the theorem is in proposition 44, while numbering has changed in subsequent editions.
28. Given that the edition was posthumous, it is safer not to rely too much on this. Nonetheless, it is quite evident that Tacquet analysis aims at highlighting the general validity of minimum principle even if it cannot provide a proof for refraction.

the other, there is no precise mathematical definition for the concept of "determined", so even a more generalized version of the axiom, would have made it useless for a mathematical proof.

The Reformulation of Teleological Demonstrations in the *Tentamen Anagogicum*

When returning to this problem in his *Tentamen Anagogicum* Leibniz will provide a rigorous definition for Tacquet intuitive concept of "determined", which will lead him to a new solution for the problem raised by concaves surfaces. In the beginning of the discussion, Leibniz returns to the ideas expressed in the *Unicum*, adding that the behavior of light on concave surfaces proofs that minimum principle should be regarded as a final cause, for the very fact that it grounds a behavior which is in accordance with the principle as it is applied in the simplest case that define the law for the more complex cases. If it were an efficient cause (as Leibniz believes it was in Fermat), then it should have been realized with respect to the actual surface and not to tangents. It seems that the accordance to an abstract rule of reduction is what makes the reference to an intelligent being unavoidable, otherwise an explanation in purely mechanical terms might be available[29]. In what follows, Leibniz seems at a first glance to be very close to Tacquet's position: the most determined path is followed when a minimum does not exist; this path can be a maximum and still be the simplest.[30]

29. "[…] je le monstray un jour par un echantillon, lors que je proposay le principe general d'optique, que le rayon se conduit d'un point à l'autre par la voye qui se trouve la plus aisée, à l'egard des superficies planes, qui doivent servir de regle aux autres. […] si on pretendoit l'employer comme une cause efficiente, et comme si tous les rayons possibles balancés entre eux le plus aisé l'emportoit, il faudroit considerer toute la surface telle qu'elle est, sans considerer le plan qui la touche, et alors la chose ne reussiroit pas toujours", *Tentamen Anagogicum*, GP VII, p. 273. In earlier years, before the discover of the conservation of force, Leibniz had proposed a case in which the easiest path is purely due to efficient causes. A compressed liquid pressing his way out of a container will try every direction but his action will be effective only were the path is the easiest ("secundum viam omnium possibilium facillimam"). Here, all possible directions of the effort have some sort of virtual physical existence which is what makes the easiest path a sort of efficient cause. By this virtual existence, Leibniz tries to derive that all possibles are in God's mind. (Cf. *Demonstratio quod deus omnia possibilia intelligit*, A VI 4b, p. 1353).
30. "D'autant plus que par ce moyen il se satisfait à leur egard à un autre principe qui succede au precedent et qui porte qu'au defaut du moindre, il faut se tenir au plus determiné, qui pourra estre le plus simple, lors même qu'il est le plus grand", *Tentamen Anagogicum*, GP VII, p. 274.

Yet there is more: minimum principle works in elementary cases, while the choice of the most determined is always respected. Therefore, there is a more general property that grounds teleological demonstrations, given by the concept of the most determined, which is still the simplest and has the easiest, conceived as minimum, as one of its special cases. Universal application of the most determined is based on the fact that maximum and minimum are obtained by the same operation, because they share the mathematical property of being what we now call extreme point. There is a tool to be employed in optical demonstrations which is independent by the fact that the varying quantity that describes the set of possible solutions is minimized or maximized. This brings Leibniz's optical demonstrations to a new and striking level of generality. Within the history of use of (proto) variational principles, by means of the concept of the most determined Leibniz treats together the various types of surfaces (rectilinear, concave and convex) and does not need to split the demonstrations. A second level of generalization is the definition of two theorems which are common to catoptric and dioptric, that geometrically describes the behavior of a "broken ray". In these theorems, reflection appears as a special case of refraction characterized by the fact that the broken ray travels in only one media.

As said, this level of generalization is given by the notion of the most determined and *prima facie* by the fact the most determined can be either a minimum or a maximum. However, in the description given in the *Tentamen*, this seem to be a derivative property, while the main feature of the most determined path is given by the fact that point that individuates it has the geometrical properties "to have no twin", or better to unify the twins in one point, which means it has no symmetrical counterpart. This type of analyses is in fact described as based only "sur l'evanouissement des differences ou sur l'unicite des jumeaux reunis et nullement sur la comparaison avec toutes les autres grandeurs"[31]. This description testifies of a change in the underlying metaphysical interpretation, connected to what is the relevant part of the process of divine choice. At the end of the *Tentamen* Leibniz relies on an example he used several times: if asked to choose among all isoperimetric triangle, divine wisdom will surely produce an equilateral triangle. In a previous version of this example a specification is made explicit: "Si nulla sit ratio

31. *Ibid*, p. 275.

specialiter agenda"[32]. Adding more requisite might imply a different outcome, which also means that the choice of the most determined is the simplest, for less requisites to determine the object must be specified: the rationality of the object of the choice comes together with the rationality of choice itself. For this reason, the simplest choice, determined by some specific properties of the object, gives a unique solution to a problem even when only *demi-détermination géométrique*. Given the assumption of the plurality of possible natural laws, deriving the law of nature can be compared to a geometrical problem of this sort: while a unique solution cannot be derived by the structure of matter and the nature of motion alone, the assumption of rational agency univocally determines laws.[33] The absolute character to the method of final causes is here expressed more generally: the analysis of the mechanism of choice, being situated at level of comparison of possibilities can solve undetermined problems, while in efficient causation models something more has to be added in order to provide a unique solution.

Since it refers directly to the structure of meta-description, the example of twins seems to me to imply more and makes much more evident that the type of restriction needed to account for a choice different then the most determined would be purely arbitrary. From this point of view, the stress is here fully on the structure of choice, more than on its outcome considered as a goal. Each possible object of choice is in itself determined enough to be realized, but at the highest level of order comparison, there is an indetermination that produces undecidability, at least among any pair of twins. Given that the meta-description as well as the relevant variables can be spatially represented, one can consider this lack of determination as expressed by the very fact that all points except the most determined can be distinguished by their symmetrical counterpart only presupposing some reference frame. Considering relevant variables, any choice among one of the twins will be purely arbitrary. Therefore, undeterminability given by symmetry suggests that the stress on the uniqueness of the most determined and

32. *De necessitate eligendi optimum* (A VI 4, p. 1351).

33. At this more general level, the addition needed to univocally determine the solution of problems in explanation by efficient causes may be attributed to elasticity, which is needed to derive fundamental laws of nature and yet cannot be deduced from the essence of matter. On its role and its evolution in Leibniz see H. Breger, "Elastizität als strukturprinzip der Materie bei Leibniz", in A. Heinekamp (hrsg.), *Leibniz' Dynamica*, Stuttgart, Franz Steiner Verlag, Studia Leibnitiana Sonderheft 13, 1984, p. 112-121.

the dismiss of comparison of magnitudes, the quantitative expression of the qualitative differences in minimum strategy, means that what is relevant for grounding the derivation of law of optic is not the description of the intrinsic nature of the most determined, but the fact that all other cases will imply a purely arbitrary choice among equal possibilities, something that Leibniz explicitly denies. In other words, no assumption on the nature of the pursued goal is needed: the most determined path is followed in every case, because it is the unavoidable (and yet contingent) outcome of the process of choice: in the *Tentamen Anagogicum*, the most general teleological explanation of optical laws does not rely much on the intrinsic nature of a goal, but on the nature of choice itself, on what is implied by the fact that a choice was needed and had been done.